Equitability and MIC: an FAQ

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The original paper on equitability and the maximal information coefficient (MIC) [Reshef et al., 2011] has generated much discussion and interest, and so far MIC has enjoyed use in a variety of disciplines. This document serves to provide some basic background and understanding of MIC as well as to address some of the questions raised about MIC in the literature, and to provide pointers to relevant supporting work.

What is equitability?

Equitability is a property of measures of dependence\(^1\) that makes them useful for data exploration. A measure of dependence is equitable if it “give[s] similar scores to equally noisy relationships of different types”. This informal definition of equitability was presented in Reshef et al. [2011], and it was subsequently formalized in Reshef et al. [2014b].

There are many possible types of equitability depending on which relationships we’re talking about (e.g., only linear relationships, only periodic relationships, all functional relationships, etc.), what kind of noise is added (e.g., mean-zero or not, added to dependent variable or independent variable or both, etc.), how we measure that noise (absolute amount of added noise, \(R^2\), etc.), and what we mean by “similar”. In addition to formally defining equitability, Reshef et al. [2014b] discusses these variations and defines several of them specifically.

One method can be either more or less equitable than another if the scores it gives to equally noisy relationships of different types are either more or less similar to each other. That is, equitability is not a binary property that a method either has or does not have — the relevant question is “how equitable is a method compared to others?”

A stronger variant of the definition of equitability is perfect equitability. A measure of dependence is perfectly equitable if it gives equal scores to equally noisy relationships of different types. So in contrast to the more general definition, perfect equitability is a binary property; in practice, however, perfect equitability may be out of reach in some settings [Kinney and Atwal, 2014], and we seek practical, efficient means for the more general, approximate notion of equitability.

When is equitability useful?

Equitability can be useful for data exploration, when we want to quickly identify the strongest associations in a data set with a large number of potentially significant relationships of unknown

\(^1\)Different people mean different things by “measure of dependence.” A simple example to hold in your head that won’t significantly compromise your reading of this FAQ is the Pearson correlation coefficient. Be aware however that in the literature we and others use the term specifically to refer to a method that gives a score of zero if the variables being analyzed are statistically independent and a non-zero score otherwise.
form. If we have an equitable statistic, we can compute it on, e.g., all the variable pairs in a data set, then rank them by their scores and examine only the top of list with the assurance that we are looking at the strongest relationships in the data set regardless of relationship type. In contrast, if we do this with a method that systematically assigns higher scores to, say, linear relationships than to others, then the top of our list may be crowded out by noisy linear relationships, causing us to potentially miss a strong non-linear relationship because of its lower score.

On the other hand, there are many situations in which equitability is either not the right goal, or not a realistic goal. If you’re trying to decide whether equitability is something you should care about when designing your analysis, we suggest the procedure in Figure 1 below.

![Equitability Diagram](image)

Figure 1: When is equitability a useful goal?

**What is MIC?**

The maximal information coefficient (MIC) is a statistic that we introduced in [Reshef et al., 2011]. It is a measure of dependence that we showed behaves more equitably than other methods in many important and common cases, such as the case of finite samples from a variety of noisy functional relationships, with noise measured by \( R^2 \). Recent uses of MIC have included: ranking viruses as candidates for rational polyvalent vaccine development [Anderson et al., 2012]; formulating a model for automatic classification of patients into disease subtypes based on their medical records [Lin et al., 2012]; understanding how pregnancy affects the relationships between groups of bacteria in the microbiome [Koren et al., 2012]; finding non-coding RNA’s involved in gene expression regulation [Qu and Adelson, 2012]; studying activity level of gut microbiota in response to behavioral changes [Maurice et al., 2013]; and identifying non-linear dynamics among social determinants of health and gonorrhea diagnosis statistics [Moonesinghe et al., 2012].

In Reshef et al. [2014b], we further develop the theory behind MIC and introduce a new statistic, MIC\(_e\), that is easier to compute and performs better than MIC in terms of equitability,
What is the statistical power of MIC?

This is an important question, but we first must ask: power against what? Conventionally, measures of dependence have focused on maximizing their power to reject a null hypothesis of statistical independence (i.e., no effect whatsoever). However, this may not be the only goal if we are exploring a data set with many very weak (but significant) effects and some potentially stronger effects. For instance, we may be interested not in identifying as many non-trivial relationships as possible, but rather in distinguishing the stronger ones from the weaker ones in order to obtain a manageable list of relationships to follow up with further analysis. In this case, our null hypothesis is not that the effect size is 0 but rather that it is small. In Reshef et al. [2014b], we prove that power against null hypotheses of this latter form is actually equivalent to equitability. Thus, equitability can be thought of as a generalization of power against independence.

What does this really mean? It means that equitability refers to how well a measure of dependence can be used to measure effect size, and not just to test for independence. Therefore, focusing solely on power against independence is not the right way to think about the utility of a measure of dependence whose goal is equitability.

Nevertheless, if all other things are equal, more power against independence is of course always better than less, so how well powered is MIC for the traditional task of rejecting a null hypothesis of statistical independence? In 2012, Simon and Tibshirani [2012] found that MIC is not as well powered as other methods. However, in an upcoming companion paper to Reshef et al. [2014b], we actually find that if power against independence is the desired goal, then simply running MIC with different parameters is enough to dramatically improve its performance. We also show that the new statistic MIC$_e$ that we introduce in Reshef et al. [2014b] has much improved power against independence compared to MIC.

For a formal discussion on the relationship between statistical power and equitability, see Section 3 of Reshef et al. [2014b].

Some people claim that “equitability” is impossible. Why do they say that?

There is a recent result that shows that perfect equitability is impossible under a specific noise model [Kinney and Atwal, 2014].

While this result is interesting and advances our understanding of equitability, its scope is very limited. First, the impossibility result holds only for perfect equitability, whereas in practice the more useful notion is the approximate notion that we introduced. To borrow an analogy from computer science, just because a problem is proven to be NP-hard does not mean that we should not pursue good approximation algorithms and heuristics for solving it.

Second, the result assumes an extremely permissive noise model in which, e.g., the mean of the noise distribution can depend on the value of the dependent variable. As others have pointed out in a technical comment [Murrell et al., 2014] on Kinney and Atwal’s paper, this model leads to identifiability issues: for instance, one can obtain the relationship $y = x^2$ as a noisy version of the relationship $y = x$. The more permissive a noise model is, the harder it is to achieve perfect equitability on it, and so an impossibility result for this noise model does not rule out the existence of perfectly equitable statistics on more restrictive models such as those we defined and
analyzed in Reshef et al. [2011], Reshef et al. [2013], and Reshef et al. [2014b]. To return to the computer science analogy: just because a problem is proven to be NP-hard does not mean that we should not try to solve it — even exactly — in special cases.

For more on this question, see our technical comment [Reshef et al., 2014a] on Kinney and Atwal’s paper, as well as Section 2.3.2 of Reshef et al. [2014b] for a more formal discussion.

**Why not just use mutual information estimation? Is mutual information estimation more equitable in general than MIC?**

Some recent work by Kinney and Atwal appears to suggest that mutual information is more equitable than MIC. A complete examination shows that this is not the case. These authors present results for only a single noise model and x-axis marginal distribution, and only at a very large sample size \(n = 5000\). In this particular instance, mutual information estimation is more equitable than MIC (albeit only slightly). However, even at this large sample size MIC substantially outperforms mutual information estimation under the vast majority of other noise types/marginals that we have analyzed; and at smaller sample sizes \((n = 250, n = 500)\) MIC substantially outperforms mutual information estimation under all the noise types/marginals that we have analyzed. We have actually presented these results — including the same analysis done by Kinney and Atwal — in our previous work. (See, e.g., Figures 6 and 7 of Reshef et al. [2013]; also see Reshef et al. [2011].) Taken together, they show that the equitability of MIC is far more robust both to sample size and to type of noise/marginal than the equitability of mutual information estimation. In summary, while one could use mutual information as an equitability score in some limited cases, our work shows that MIC will yield better results on most data sets.

For more on this, see our technical comment [Reshef et al., 2014a] on Kinney and Atwal’s paper, as well as the analyses conducted in Reshef et al. [2011, 2013]. There is also an upcoming companion paper to Reshef et al. [2014b] that conducts more detailed quantitative comparisons that confirm these conclusions.

**Should I use MIC in my data analysis?**

Perhaps. First, consult Figure 1 to see if it makes sense to make equitability a goal of yours. If it does, you should probably use MIC\(^2\) due to the robustness of its equitability. The one exception to this rule is if both of the following conditions hold: a) your sample size is very large \((n \gtrsim 5000)\) and b) you are confident that your noise model involves Gaussian noise only in the dependent variable and no noise at all in the independent variable. In this case, mutual information estimation has slightly improved equitability over MIC. For more detailed information, see, e.g., Figures 6 and 7 of Reshef et al. [2013] and Figure 1 of Reshef et al. [2014a], as well as Reshef et al. [2011].

\(^2\)We are not yet explicitly recommending the use of MIC\(_e\) because the analyses demonstrating its superior performance are still unpublished. However, in the future we expect that it will be used in place of MIC due to its improved equitability, power against independence, and speed.
References


